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Non-equilibrium statistical aspects of fatigue fracture of epoxy matrix composites

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The non-equilibrium statistical fracture theory has been used to study the statistical aspects of fatigue fracture of fibre-reinforced epoxy composites. An equation for the evolution of microcracks has been established and the general behaviour of nucleation and growth of microcracks discussed. Subsequently, the distribution function of crack density, the fracture probability, the reliability and the *S–N* relation for the composites have been derived. The theoretical results obtained coincide qualitatively with experiments.

1. Introduction

Much theoretical and experimental work has been performed recently on the statistical nature of the fatigue fracture of composites [1–3]. In general, these studies are mostly phenomenological and only in a few cases has the statistical nature of one problem in the fatigue of composites been studied, for example, the statistical nature of fatigue life or fatigue strength. By fitting with experimental data, many investigators have proposed two functions to describe the distribution of fatigue life and fatigue strength of composites: the Weibull distribution function and the log-normal distribution function [3]. However, the statistical nature of different problems in the fatigue of composites reflects the different aspects of the macroscopic behaviour of the evolution of microstructure in the composites. Therefore, the statistical aspects of composites in fatigue, on the one hand, are determined by the evolution behaviour of microcracks, and on the other hand, should correlate with each other in some way.

In the present work, the non-equilibrium statistical fracture theory [4] has been used to study the statistical aspects of fatigue fracture of fibre-reinforced epoxy composites on a unified physical model: the stochastic model of crack growth. An equation for the evolution of a fatigue crack has been established. The experimental law for the nucleation and growth of fatigue cracks in the composites is discussed and the distribution function of crack density, the fracture probability, the reliability and the *S–N* relation for the composites have subsequently been derived.

2. Physical aspects and the evolution equation

Experiments show that the process of fatigue fracture of a composite is a process in which microcracks nucleate, then grow continuously in the composite under the action of cyclic stress; eventually one of the cracks propagates at a high rate, causing catastrophic failure of the composite. The growth rate of fatigue cracks, $\dot{c}_N$, is typically described by [5]

$$\dot{c}_N = \frac{dc}{dN} = B\beta(c) \quad (1)$$

where c is the crack length, N is the number of cycles of applied cyclic stress, B is a constant depending on the material characteristics, the applied cyclic stress, the direction of crack growth, temperature and moisture, $\beta(c)$ is a function of crack length. The conventional approach to predict fatigue fracture is to integrate Equation 1 to obtain the lifetime of materials [6]. However, as is well known, owing to the inhomogeneity of defects and compositions in real materials, their microstructure is always inhomogeneous. This structural inhomogeneity leads to a fluctuation in the microcrack growth rate and the statistical nature of the fatigue law. The fluctuation of the crack growth rate is stochastic and independent of the crack growth history. Hence the conventional approach is inadequate, because it neglects the crack statistics and only considers growth of a single crack, and the statistical approach must be employed. When the stochasticity of crack growth is considered, Equation 1 must be a stochastic equation. Consequently, the crack length, c, is a random variable and B is a random parameter. For convenience in handling, we regard the microstructure of a material as an average structure background being superimposed by inhomogeneous fluctuation, owing to all kinds of inhomogeneity. In this case, the random parameter, B, changes to $B + f(N)$, where $f(N)$ is a fluctuation function, so that the crack growth rate has the following form

$$\dot{c} = B\beta(c) + \beta(c)f(N) \quad (2)$$

where $B\beta(c)$ is the transport growth rate which is determined by the average structure background and the applied cyclic stress, $\beta(c)f(N)$ is the fluctuation growth rate which is determined by inhomogeneous

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fluctuation of the material structure and the applied cyclic stress. For the convenience of calculation, we suppose the process of crack growth to be a Markov process and the fluctuation function $f(N)$ to obey the Gaussian distribution which satisfies

$$\langle f(N) \rangle = 0$$
$$\langle f(N)f(N') \rangle = D\delta(N - N') \quad (3)$$

where δ is the Dirac function and D is the fluctuation growth coefficient. According to stochastic theory, the Fokker–Planck equation, which is equivalent to Equation 1, is [7]

$$\frac{\partial p(c_0, cN)}{\partial N} = -\frac{\partial}{\partial c}\left\{\left[B\beta(c) + \frac{D}{2}\beta(c)\frac{\partial \beta(c)}{\partial c}\right]p(c_0, c, N)\right\} + \frac{D}{2}\frac{\partial^2}{\partial c^2}\{\beta^2(c)p(c_0, c, N)\} \quad (4)$$

where $p(c_0, c, N)\mathrm{d}c$ is the probability that a crack grows from the initial length, c_0, to the length between c and $c + \mathrm{d}c$ under the action of cyclic stress for N cycles. Similarly, when a large number of cracks is considered, the evolution equation of fatigue cracks is established.

$$\frac{\partial M(c, N)}{\partial N} = -\frac{\partial}{\partial c}\left\{\left[B\beta(c) + \frac{D}{2}\beta(c)\frac{\partial \beta(c)}{\partial c}\right]M(c, N)\right\} + \frac{D}{2}\frac{\partial^2}{\partial c^2}[\beta^2(c)M(c, N)\} + q(N)\delta(c - c_0) \quad (5)$$

where $M(c, N)\mathrm{d}c$ is the density of cracks evolving in the length between c and $c + \mathrm{d}c$ under the action of cyclic stress for N cycles, $q(N)$ is the nucleation rate of microcracks. Considering that there is no crack in the material at time zero and no crack with infinite length exists in materials, we can determine the initial and boundary conditions as follows.

$$M(c, N = 0) = 0, \qquad M(c \to \infty, N) = 0 \quad (6)$$

From Equation 5 it can be seen that the rate of change of the distribution function of the crack density is determined by the transport growth rate, the fluctuation growth rate and the nucleation rate. If B, $\beta(c)$ $q(N)$ and D are known, the distribution function of crack density can be obtained by solving Equations 5 and 6.

3. Nucleation rate, growth rate and fluctuation growth coefficient

3.1. Nucleation rate

For the fibre reinforced epoxy composites, the fatigue cracks mostly nucleate at the interfaces between the fibres and the matrix by debonding [8]. Compared with the case of metals, a much smaller part of the fatigue life is spent in crack nucleation in composites. Hence, for simplicity, here we adopt the phenomenological treatment rather than discuss the micromechanism of crack nucleation.

We define M_0 as the average density of latent nuclei of cracks and α as the probability for such a nucleus to form a crack. α is dependent on materials characteristics, the applied cyclic stress, and crack orientation. We assume that cracks do not nucleate at locations where cracks have formed. Then we obtain the differential equation for crack density function, $M(N)$, as follows

$$q(N) = \frac{\mathrm{d}M(N)}{\mathrm{d}N} = \alpha[M_0 - M(N)] \quad (7)$$

Considering that $M(N = 0) = 0$, we obtain the nucleation rate of cracks as

$$q(N) = \alpha M_0 \exp(-\alpha N) \quad (8)$$

3.2. Growth rate

Many experiments show that for many composites the fatigue crack growth rate can be described by the Paris formula [9–11]

$$\frac{\mathrm{d}c}{\mathrm{d}N} = A(\Delta K)^n (\mathrm{m\,cycle}^{-1}), \; \Delta K = \sigma_a Y c^{1/2}\, (\mathrm{MPa\,m}^{1/2}) \quad (9a)$$

where ΔK is the stress intensity range, σ_a is the stress amplitude, and Y is a geometric factor; A and n are material constants depending on material characteristics (such as fibre orientation, stress state, crack orientation and environment). For the crack propagation in Mode II loading by delamination, starting from the damage zone in the outer layers of the specimen and propagating towards the ends of the unidirectional E glass fibre/epoxy specimen in a bending experiment with the cyclic load perpendicular to the fibre orientation, $n = 9$ and $A = 1.6 \times 10^{-12}$ [9]. In comparison with Equation 1, Equation 9a can be rewritten in another form

$$\frac{\mathrm{d}c}{\mathrm{d}N} = B\beta(c) \quad (9b)$$

where

$$B = A(\sigma_a Y)^n, \quad \beta(c) = c^{n/2} \quad (10)$$

3.3. Fluctuation growth coefficient

The fluctuation growth coefficient has been derived theoretically [4]. For the composites discussed here, we have deduced

$$D = B^2\eta^2 = A^2(\sigma_a Y)^{2n}\eta^2 \quad (11)$$

where η is a dimensionless constant, which represents the effect of the fluctuations of material and circumstantial parameters included in A and n on the fluctuation of the crack growth rate.

4. Distribution function of crack density

Substituting Equations 8, 10 and 11 into Equation 5 we obtain the distribution function of crack density

$$M(c,N)\,\mathrm{d}c = \int_0^N \frac{q(N')}{[2\pi D(N-N')C^n]^{1/2}} \times \exp\left\{-\left[\frac{2}{n-2}\left(\frac{1}{c_0^{\frac{n}{2}-1}} - \frac{1}{c^{\frac{n}{2}-1}}\right) - B(N-N')\right]^2 \Big/ 2D(N-N')\right\} \mathrm{d}N'\,\mathrm{d}c \quad (12)$$

and the probability of finding the crack length between c and $c + \mathrm{d}c$ at N cycles

$$p(c,N)\,\mathrm{d}c = \frac{M(c,N)\,\mathrm{d}c}{\int_0^\infty M(c,N)\mathrm{d}c} = \frac{\alpha}{1-\mathrm{e}^{-\alpha N}} \int_0^N \frac{\mathrm{e}^{-\alpha N'}}{[2\pi D(N-N')c^n]^{1/2}} \times \exp\left\{-\left[\frac{2}{n-2}\left(\frac{1}{c_0^{\frac{n}{2}-1}} - \frac{1}{c^{\frac{n}{2}-1}}\right) - B(N-N')\right]^2 \Big/ 2D(N-N')\right\} \mathrm{d}N'\,\mathrm{d}c \quad (13)$$

Obviously it is normalized to $\int_0^\infty p(c,N)\mathrm{d}c = 1$. From the above two equations we can see that the distribution function of crack density, $M(c, N)$, and the probability of density function of microcracks, $p(c, N)$, are determined by the microscopic behaviour of nucleation and growth of fatigue cracks, which are represented by $q(N)$, B, $\beta(c)$ and D.

For the sake of simplicity of analytical derivation we adopt the following approximate form of $p(c, N)$

$$p(c,N)\,\mathrm{d}c \simeq \frac{1}{(2\pi D N c^n)^{1/2}} \times \exp\left\{-\left[\frac{2}{n-2}\left(\frac{1}{c_0^{\frac{n}{2}-1}} - \frac{1}{c^{\frac{n}{2}-1}}\right) - BN\right]^2 \Big/ 2DN\right\} \mathrm{d}c \quad (14)$$

This can be converted to the probability of finding crack length c in the cycle number interval $(N, N + \mathrm{d}N)$, $p(N, c)\mathrm{d}N$ [4]

$$p(N,c)\,\mathrm{d}N = \frac{B}{(2\pi D N)^{1/2}} \times \exp\left\{-\left[\frac{2}{n-2}\left(\frac{1}{c_0^{\frac{n}{2}-1}} - \frac{1}{c^{\frac{n}{2}-1}}\right) - BN\right]^2 \Big/ 2DN\right\} \mathrm{d}N \quad (15)$$

5. Fracture probability and reliability

As discussed in Section 2, when cracks grow to a definite length under the action of cyclic stress, one of them propagates at a higher rate, causing catastrophic failure of material. However, for composites the fracture criterion has not been well defined. Hence here we only suppose that there exists a critical length, c_f, beyond which a crack may propagate in an unstable manner.

From Equation 15 we can determine the probability of finding a crack which loses its stability under the action of cyclic stress for cycles between N and $N + \mathrm{d}N$

$$p(N,c_f)\,\mathrm{d}N = \frac{B}{(2\pi D N)^{1/2}} \times \exp\left\{-\left[\frac{2}{n-2}\left(\frac{1}{c_0^{\frac{n}{2}-1}} - \frac{1}{c_f^{\frac{n}{2}-1}}\right) - BN\right]^2 \Big/ 2DN\right\} \mathrm{d}N \quad (16)$$

According to the principle of minimum strength, the fatigue fracture probability, P_f, of the composite under the action of cyclic stress for cycles between 0 and N is

$$P_f(N) = 1 - \left[1 - \int_0^N p(N,c_f)\,\mathrm{d}N\right]^{M_1} \simeq 1 - \exp\left[-M_1 \int_0^N p(N,c_f)\,\mathrm{d}N\right] \quad (17)$$

where $M_1 = M(N)V$ is the total number of cracks, V is the volume of the material. The approximate expression on the right-hand side of Equation 17 comes from assuming that $\int_0^N p(N,c)\,\mathrm{d}N \ll 1$. Because $\int_0^\infty p(N,c)\,\mathrm{d}N = 1$, obviously $P_f(N=0) = 0$ and $P_f(N \to \infty) = 1$. The fracture probability, W_f, of the material under the action of cyclic stress with amplitude σ_a for cycles between N and $N + \mathrm{d}N$ is

$$W_f(N)\,\mathrm{d}N = \frac{\mathrm{d}P_f(N)}{\mathrm{d}N}\mathrm{d}N = M_1\left[1 - \int_0^N p(N,c_f)\,\mathrm{d}N\right]^{M_1-1} p(N,c_f)\mathrm{d}N \simeq M_1 \exp\left[-M_1 \int_0^N p(N,c_f)\,\mathrm{d}N\right] p(N,c_f)\,\mathrm{d}N \quad (18)$$

Similarly, from Equation 14 we can also obtain the probability of finding a crack losing its stability under the action of cyclic stress with amplitude between σ_a and $\sigma_a + \mathrm{d}\sigma_a$ for N cycles, $p(\sigma_a, N)\mathrm{d}\sigma_a$

$$p(\sigma_a,N)\,\mathrm{d}\sigma_a = p(c_f,N)\left|\frac{\mathrm{d}C_f}{\mathrm{d}\sigma_a}\right|\mathrm{d}\sigma_a = \frac{1}{(2\pi D N C^n)^{1/2}} \exp\left\{-\left[\frac{2}{n-2} \times \left(\frac{1}{c_0^{\frac{n}{2}-1}} - \frac{1}{c_f^{\frac{n}{2}-1}}\right) - BN\right]^2 \Big/ 2DN\right\} \left|\frac{\mathrm{d}c_f}{\mathrm{d}\sigma_a}\right| \mathrm{d}\sigma_a \quad (19)$$

Then the fracture probability of the composite under the action of cyclic stress with amplitude between 0 and σ_a for N cycles is

$$P_f(\sigma_a) = 1 - \left[1 - \int_0^{\sigma_a} p(\sigma_a,N)\mathrm{d}\sigma_a\right]^{M_1} \simeq 1 - \exp\left[-M_1 \int_0^{\sigma_a} p(\sigma_a,N)\,\mathrm{d}\sigma_a\right] \quad (20)$$

The approximate expression on the right-hand side of Equation 20 comes from assuming that $\int_0^{\sigma_a} p(\sigma_a,N)\,\mathrm{d}\sigma_a \ll 1$. Because $\int_0^\infty p(\sigma_a,N)\mathrm{d}\sigma_a = 1$, obviously $P_f(\sigma_a = 0) = 0$ and $P_f(\sigma_a \to \infty) = 1$. The corresponding fracture probability density function is

$$W_f(\sigma_a)d\sigma_a = \frac{dP_f(\sigma_a)}{d\sigma_a}d\sigma_a$$
$$= M_1\left[1 - \int_0^{\sigma_a} p(\sigma_a, N)d\sigma_a\right]^{M_1-1} p(\sigma_a, N)d\sigma_a$$
$$\simeq M_1 \exp\left[-M_1 \int_0^{\sigma_a} p(\sigma_a, N)d\sigma_a\right] p(\sigma_a, N)d\sigma_a \quad (21)$$

The reliability, R, of a material is defined as the probability that the material does not fracture. According to the probability theory the reliability of the composite under the action of cyclic stress with amplitude σ_a for cycles between 0 and N is

$$R(N) = 1 - P_f(N)$$
$$= \left[1 - \int_0^N p(N, c_f)dN\right]^{M_1}$$
$$\simeq \exp\left[-M_1 \int_0^N p(N, c_f)dN\right] \quad (22)$$

and the reliability of the composite under the action of cyclic stress with amplitude between 0 and σ_a for N cycles is

$$R(\sigma_a) = 1 - P_f(\sigma_a)$$
$$= \left[1 - \int_0^{\sigma_a} p(\sigma_a, N)d\sigma_a\right]^{M_1}$$
$$\simeq \exp\left[-M_1 \int_0^{\sigma_a} p(\sigma_a, N)d\sigma_a\right] \quad (23)$$

The above two equations are the distribution functions of fatigue life and fatigue strength of the composite, respectively. Now we can clearly see that the fracture probabilities and the reliabilities as functions of cycles and stress amplitude just represent two different aspects of the statistical nature of a composite under fatigue. Their forms are determined by the evolution behaviour of microcracks.

In order to obtain the concrete forms of fracture probability and reliability, we have to calculate the two integrations $\int_0^N p(N, c_f)dN$ and $\int_0^{\sigma_a} p(\sigma_a, N)d\sigma_a$. We first write them in another form

$$p(N, c)dN = \frac{1}{\pi^{1/2}}\int_X^{\infty} \exp(-X^2)\left|\frac{dX}{dN}\right| dN \quad (24)$$

$$p(\sigma_a, N)d\sigma_a = \frac{1}{\pi^{1/2}}\int_X^{\infty} \exp(-X^2)\left|\frac{dX}{dc_f}\frac{dc_f}{d\sigma_a}\right| d\sigma_a \quad (25)$$

where

$$X = \frac{2}{n-2}\left(\frac{1}{c_0^{\frac{n}{2}-1}} - \frac{1}{c_f^{\frac{n}{2}-1}}\right) - BN \Big/ (2DN)^{1/2} \quad (26)$$

For the convenience of derivation, the following approximate formula is used

$$\frac{2}{\pi^{1/2}}\int_X^{\infty} \exp(-X^2)dX = \frac{1}{aX^8} \quad a = 0.8, X > 1.5 \quad (27)$$

Substituting Equations 24 and 25 into Equations 16, 17 and 20–23, we can derive the concrete forms of fracture probability and reliability.

$$R(N) = 1 - P_f(N) = \exp\left\{-\left(\frac{M(N)V}{2a}\right)\times\left[2^{1/2}A(\sigma_a Y)^n\eta \Big/ \frac{2}{n-2}\left(\frac{1}{c_0^{\frac{n}{2}-1}} - \frac{1}{c_f^{\frac{n}{2}-1}}\right)\right]^8 N^4\right\} \quad (28)$$

$$W_f(N) = \frac{2M(N)V}{a}\left[2^{1/2}A(\sigma_a Y)^n\eta \Big/ \frac{2}{n-2}\times\left(\frac{1}{c_0^{\frac{n}{2}-1}} - \frac{1}{c_f^{\frac{n}{2}-1}}\right)\right]^8 N^3 R(N) \quad (29)$$

$$R(\sigma_a) = 1 - P_f(\sigma_a) = \exp\left\{-\left(\frac{M(N)V}{2a}\right)\times\left[(2N)^{1/2}AY^n \Big/ \frac{2}{n-2}\left(\frac{1}{c_0^{\frac{n}{2}-1}} - \frac{1}{c_f^{\frac{n}{2}-1}}\right)\right]^8 \sigma_a^{8n}\right\} \quad (30)$$

$$W_f(\sigma_a) = \frac{4nM(N)V}{a}\left[(2N)^{1/2}AY^n \Big/ \frac{2}{n-2}\times\left(\frac{1}{c_0^{\frac{n}{2}-1}} - \frac{1}{c_f^{\frac{n}{2}-1}}\right)\right]^8 \sigma_a^{8n-1}R(\sigma_a) \quad (31)$$

Equations 28 and 30 are the two-parameter Weibull distribution functions of fatigue life and fatigue strength as follows [11]

$$R(N) = \exp\left[-\left(\frac{N}{N_r}\right)^{\beta}\right] \quad (32)$$

and

$$R(\sigma_a) = \exp\left[-\left(\frac{\sigma_a}{\sigma_r}\right)^{\gamma}\right] \quad (33)$$

where β, γ are the shape parameters and N_r, σ_r are the scale parameters. These parameters are usually determined experimentally, while in our theory the parameters are determined theoretically by the microbehaviour of crack evolution, applied cyclic

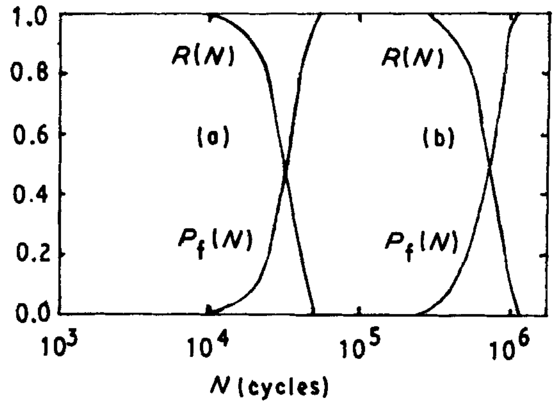

Figure 1 Fracture probability $P_f(N)$, and reliability, $R(N)$, for different stress amplitudes: (a) $\sigma_a = 630$ MPa, (b) $\sigma_a = 530$ MPa.

stress and the material characteristics. For the unidirectional, continuous E glass fibre/epoxy composite under bending experiment mentioned in Section 3, these functions are plotted in Figs 1–4. When $\sigma_{max} = 700$ MPa and the stress ratio $R = 0.1$, the shape and scale parameters of the distribution function of fatigue life are calculated to be 4.00 and 37 607, while the experimental results are 3.34 and 40 100, respectively. The values of the parameters used in our calculation are: $n = 9$, $A = 1.6 \times 10^{-12}$, $V = 3.8 \times 10^{-2}\,\mathrm{m}^3$, $M_0 = 10^4\,\mathrm{m}^{-3}$, $\alpha = 0.1$, $c_0 = 4 \times 10^{-6}\,\mathrm{m}$, $Y = 1.772$, $\eta = 1.0$, $c_f = 10^{-3}\,\mathrm{m}$.

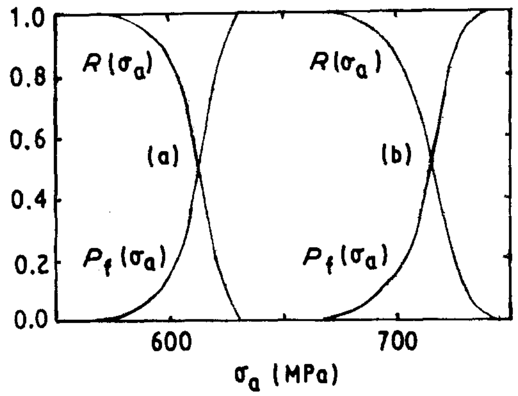

Figure 2 Fracture probability, $P_f(\sigma_a)$, and reliability, $R(\sigma_a)$, for different cycles: (a) $N = 5 \times 10^4$, (b) $N = 10^4$.

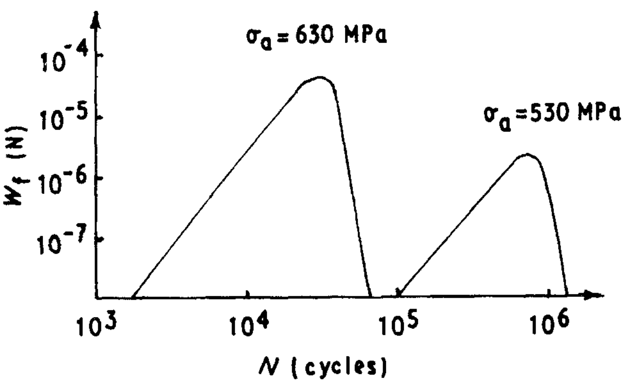

Figure 3 Fracture probability density, $W_f(N)$, for different stress amplitudes.

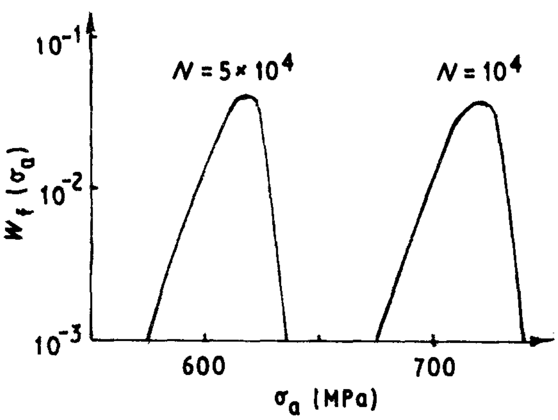

Figure 4 Fracture probability density, $W_f(\sigma_a)$ for different cycles.

6. *S–N* relation

According to statistical mechanics, the average life of the composite is

$$N_f = \int_0^{\infty} N W_f(N)\,dN = \int_0^{\infty} R(N)\,dN \qquad (34)$$

Substituting Equation 28 into the above equation and considering that $M(N)$ changes slowly with N, we obtain

$$N_f = \left(\frac{2a}{M(N_f)V}\right)^{1/4} \Gamma\left(\frac{5}{4}\right) \times \left[\frac{2}{2^{1/2} A (\sigma_a Y)^n (n-2)} \left(\frac{1}{c_0^{\frac{n}{2}-1}} - \frac{1}{c_f^{\frac{n}{2}-1}}\right)\right] \qquad (35)$$

It can be seen that the fatigue life decreases with increasing stress amplitude. The size effect is also included in Equation 35. The calculated *S–N* curve is shown in Fig. 5.

Equation 35 can be rewritten in another form.

$$\sigma_a N^m = b$$

where $m = 1/2n$ and

$$b = \left\{\left(\frac{2a}{M(N)V}\right)^{1/4} \Gamma\left(\frac{5}{4}\right) \times \left[\left(\frac{1}{c_0^{\frac{n}{2}-1}} - \frac{1}{c_f^{\frac{n}{2}-1}}\right) \Big/ 2^{1/2}(n-2)AY^n\right]^n\right\}^{1/2n} \qquad (36)$$

which coincides with the empirical formula of the *S–N* relation for the composite [1]. Because of lack of corresponding experimental data for comparison, this result can only serve as a prediction.

7. Conclusion

The non-equilibrium statistical fracture theory has been used to study the statistical nature of fatigue fracture of composites. The two-parameter Weibull distribution functions of fatigue life and fatigue strength for the composites are derived theoretically on a unified physical model. These functions may all be expressed by the same set of physical quantities as is used to express the nucleation and growth rates of

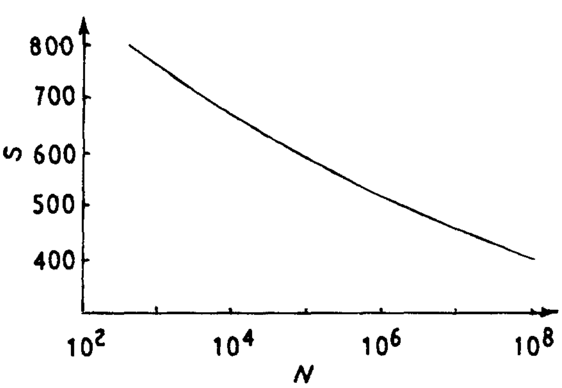

Figure 5 Calculated *S–N* curve for the unidirectional E glass fibre/epoxy composite.

microcracks. The theoretical results obtained not only explain some experimental data qualitatively but also give some predictions of the statistical properties of composites in fatigue.

Because the mechanisms of nucleation and growth of microcracks are not well understood, the effects of material characteristics, stress state, crack orientation and environment on the crack growth rate, which are all included in the parameters A, n and Y, are not expressed analytically. Therefore, in order to obtain analytic relations between the statistical results derived in this paper and the various factors just mentioned, further discussion of the mechanisms of nucleation and growth of fatigue microcracks is required.

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